- Consider the following performance surface:

$$F(X) = 5x_1^2 + 4x_2^2 - 6x_1x_2 - 3x_1 + 2$$

a. Find the gradient of this of this function at point **(3, 1)**

b. Find the Hessian of this of this function at point **(3 , 1)**

- Consider the following performance surface:

$$\boldsymbol{F(X) = x_1^2 x_2 x_3}$$

Find the second order Taylor series expansion of this function around point $P = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$

- Consider the following performance surface:

$$\boldsymbol{F(X) = 4x_1^2 + 7x_2^2 + 4x_1x_2}$$

  - Find the stationary point of this surface
  - Determine if the stationary point is strong minima, weak minima, strong maxima, or weak maxima. Show your calculations and explain your conclusion.

- Consider the following performance surface:

$$F(X) = 4x_1^2 - 2x_2^2 + 3x_1x_2 - 2x_1 + 6$$

Calculate the first and second directional derivative of this function at point **(1,2)** in the direction of **(8,6)**

First directional derivative=

Second directional derivative=

Consider the following performance surface:

$$F(X) = x_1^3 + 3x_1x_2 - 5x_1 + 8$$

Calculate the **FIRST** and **SECOND** directional derivative of this function at point **(2,1)** in the direction of **(3,4)**

- Consider the following performance surface:

$$\boldsymbol{F(X) = x_1^3 - 2x_2^2 + 5x_3^2 + 4x_1x_2}$$

Find the second order Taylor series expansion of this function around point $P = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

Note: second order means to include second order derivatives

Show both matrix version and expanded version